

## OLD AND NEW VERSIONS OF WOLF SUNSPOT NUMBERS: CONSISTENCY OF CHARACTERISTICS FOR RESTORED AND INSTRUMENTAL PARTS OF SERIES

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**Keywords:** Restored and reliable parts of WSN, Cycle group, statistical estimates, approximation

**Abstract:** The paper considers the Zurich series of monthly averaged sunspot numbers  $W$  ( $W = W_{rest} \cup W_{tool}$ ), which includes the restored  $W_{rest}$  series (from 1749 to 1849) and  $W_{tool}$ , a series of reliable Wolf numbers (regular instrumental observations from 1849 to the present). When comparing extended fragments, the local data residual plays a smaller role and more balanced estimates are obtained. Classification of cycles by duration allows us to describe the relationship between the parameters of reliable cycles and to show the inconsistency of these parameters for the 1-9 cycle group. A more realistic scenario for the  $W_{rest}$  series corresponds to a simultaneous reduction in the length of the series by 4–5 years and an increase in the “average”  $W$  by 1.25 times.

## СТАРАЯ И НОВАЯ ВЕРСИИ РЯДА ЧИСЕЛ ВОЛЬФА: СОГЛАСОВАННОСТЬ ХАРАКТЕРИСТИК ВОССТАНОВЛЕННОЙ И ИНСТРУМЕНТАЛЬНОЙ ЧАСТЕЙ РЯДОВ

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**Ключови думи:** Восстановленная и достоверная части ряда WSN, статистические оценки, группа циклов, аппроксимация

**Резюме:** Естественно желание включать столетний интервал восстановленной части ряда чисел Вольфа  $W_{rest}$  (1749–1849 гг.) в исследования наравне с инструментальной (достоверной) частью ряда  $W_{tool}$  (с 1849 г. по настоящее время). Но при объединении отрывочных данных с различными плотностью наблюдений, амплитудным разрешением и масштабированием искажаются, в первую очередь, локальные характеристики регистрируемого процесса и взаимосвязь (гладкость) временных фрагментов разного масштаба. Поэтому важно оценить степень согласованности рядов  $W_{rest}$  и  $W_{tool}$ , а также непротиворечивость параметров самого восстановленного ряда. В работе использована классификация циклов по длительности, что позволило параметризовать и аппроксимировать некоторые характеристики циклов достоверного ряда. Полученные зависимости расширили возможности анализа ряда  $W_{rest}$ .

### Introduction

The paper considers the Zurich series of monthly averaged sunspot numbers  $W$  ( $W = W_{rest} \cup W_{tool}$ ), which includes the restored  $W_{rest}$  series (from 1749 to 1849) and  $W_{tool}$ , a series of reliable Wolf numbers (regular instrumental observations from 1849 to the present). Naturally, there is a desire to include the centennial interval of the restored part of the series of Wolf numbers  $W_{rest}$  in studies along with the instrumental part of the series  $W_{tool}$ . The data reliability for the  $W_{rest}$  series and possible use of them in research is still a challenge. When combining fragmentary data [1] with

different density of observations, amplitude resolution and scaling, the local characteristics of the recorded process and the consistency of time fragments of different scales (for example, the structure of the cycles and their relationship) are distorted. Therefore, it is reasonable to research the parameters of extended fragments without detailing their “complex” history of development. One approach that evaluates the consistency of the *Wrest* and *Wtool* series is to compare the characteristics of the selected intervals [2]. For instance, the correspondence of statistical estimates of cycle basic parameters (cycle duration  $T_c$  and its growth branches  $T_m$ , cycle maximum  $W_m$ , cycle area  $S_q$ ) for two groups of cycles: a group of restored cycles 1–9 and a group of reliable cycles 10–23. The Table shows the average, the root of the variance and their ratio calculated for these groups. The upper readings correspond to the old version; the lower readings correspond to the new one. The cycle parameters in the new version of a Wolf number series are taken from paper [3]. It can be seen that the characteristics of the second group, in italics, are better, and much better for  $T_c$ ,  $T_m$ . Additionally, cycles 1, 5 and 7 have abnormally long growth branches – more than half of the cycle.

Table. Statistical Estimates of *Wrest* and *Wool* series cycles

	mean I ÷ IX / X ÷ XXIII	$\sigma^{1/2}$ I ÷ IX / X ÷ XXIII	$\sigma^{1/2} / \text{mean}$ I ÷ IX / X ÷ XXIII
<b>Tc</b>	134.33 / 131.14 134.33 / 131.14	18.95 / 10.02 18.88 / 9.48	0.141 / 0.078 0.141 / 0.072
<b>Tm</b>	56.56 / 46.43 56.67 / 47.93	17.51 / 6.39 17.67 / 7.25	0.310 / 0.138 0.312 / 0.151
<b>Wm</b>	105.59 / 119.66 175.98 / 184.92	40.16 / 38.02 66.95 / 47.75	0.380 / 0.318 0.380 / 0.258
<b>Sq</b>	6301.52 / 7329.54 10609.16 / 11341.85	2238.57 / 2115.12 3833.60 / 2532.89	0.355 / 0.288 0.361 / 0.223

For more correctness, let us compare the groups with the same number of cycles, then the group of restored cycles can be assigned six groups of nine cycles from the reliable part of the *Wtool* series: 10–18, 11–19, 12–20, 13–21, 14–22, 15–23. Let us denote these groups of reliable cycles, respectively, as G1, G2, G3, ..., G6, and the group of restored cycles as G0. An example of the ( $\sigma^{1/2} / \text{mean}$ ) ratio for the  $T_c$  and  $S_q$  parameters for all groups is shown in Fig. 1, where the readings of the new version are marked with “+”. And in this case, the time parameters of the reliable groups with close values differ from the corresponding estimates for the cycles 1–9. We also note that the estimates of the G1–G6 and G0 group area in the new version are less consistent than in the old one.

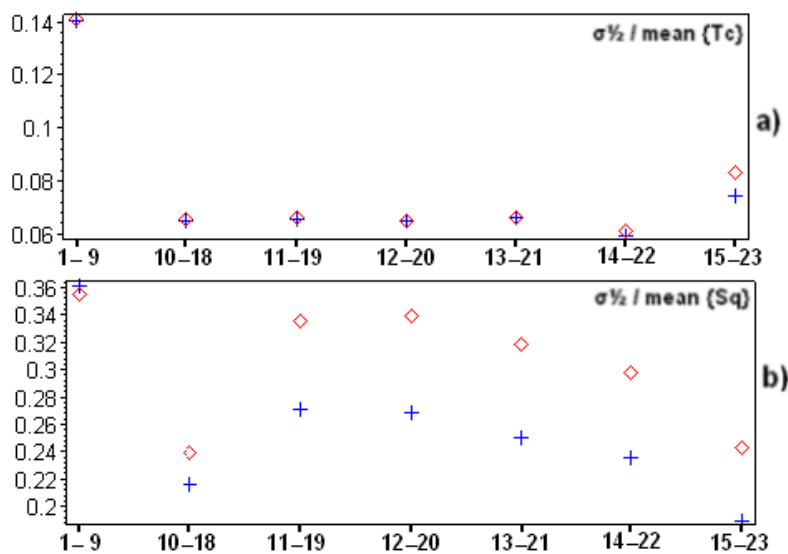


Fig. 1. Relative characteristics of  $T_c$  (a) and  $S_q$  (b) for cycle groups; “+” readings of the new Wolf number version

This work is aimed at analysing groups and comparison of their properties for the Wrest and Wtool series, in both versions of the Wolf number series. It is important to evaluate not only the consistency degree for these series, but also the consistency of the Wrest series parameters. In the analysis, we used the classification of cycles by duration: “long” cycles with  $T_c > 133$  months and “short” cycles ( $T_c < 133$  months).

### Characteristics of cycle groups for the Wrest and Wtool series

Let us rely on the characteristics of the G0+G6 groups. We associate the total duration of the  $\Sigma T$  cycles and the area of the  $\Sigma Sq$  cycles in the group with each group, let us set  $w = \Sigma Sq / \Sigma T$  for the estimate of the “average” W value in the group. For the G0 group of restored cycles, these parameters are equal:  $\Sigma T_0 = 1209$  months,  $\Sigma Sq_0 = 56713.65/95482.44$  (old/new version),  $w_0 \sim 46.91/78.98$  (old/new version). The duration of each group in months is illustrated in Fig. 2a, the horizontal line is the average  $\Sigma T$  value for G1–G6 groups equal to 1159.0 months. The numbers in parentheses (5)–(2) indicate the number of “long” cycles in this group.

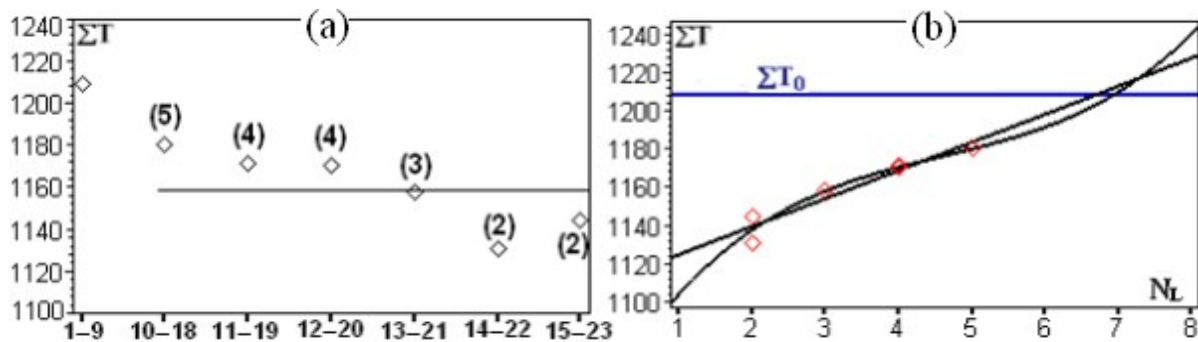


Fig. 2. Distribution of  $\Sigma T$  in cycle groups – (a); dependence of  $\Sigma T$  on the number of “long” cycles  $N_L$  in the group – (b)

It can be seen that the ratio of “long” and “short” cycles in a group adequately reflects its length and can be used to classify groups of reliable cycles. The Figure 2b shows the dependence of  $\Sigma T$  on the number of “long” cycles  $N_L$  in the group, for groups G1–G6, and the value of  $\Sigma T_0$ . The linear and cubic approximating functions are superimposed on  $\Sigma T$ . Let us denote that the  $\Sigma T$  values for both versions of the Wolf series actually coincide. For  $\Sigma Sq$ , the relationship with the number of “long” cycles  $N_L$  in the group is different for these versions, as shown by Fig. 3, where  $\Sigma Sq_0$  are marked. Quadratic approximating functions are superimposed on  $\Sigma Sq$ .

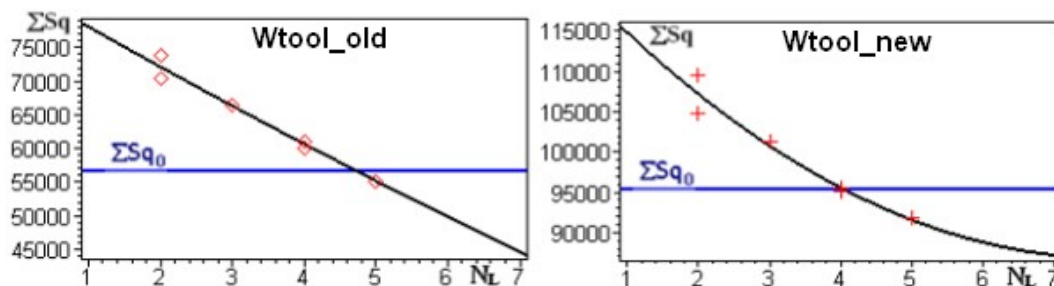


Fig. 3. Dependence of the total area of cycles in the group  $\Sigma Sq$  on the number of “long” cycles  $N_L$  in the group

### Assessment of the parameter consistency for the group of restored cycles G0

The group of restored cycles, including six “long” cycles, has the associated parameters  $\Sigma T_0$ ,  $\Sigma Sq_0$ , and  $N_L=6$ . Let us evaluate their consistency using the obtained dependences for the following three options.

V-1. We proceed from the tabular duration  $\Sigma T_0$  of the 1+9 cycles. According to the  $\Sigma T$  curves (Fig. 2b), this corresponds to  $N_L=7$  (seven “long” and two “short” cycles). Using the curves  $\Sigma Sq$  (Fig. 3) for  $N_L=7$ , we find the total area for this combination of nine cycles without detailing their sequence. For the old

version  $\Sigma Sq(N_L=7) = 45850.26$ , for the new version,  $\Sigma Sq(N_L=7)=87191.60$ . These values are much less than  $\Sigma Sq_0$  – tabular total areas of the 1–9 cycles (according to the Table, more than by the average area of the cycle). In fact, we get an even lower level of solar activity at the Dalton minimum and its surroundings, which is very exotic.

V-2. Based on the  $\Sigma Sq_0 = 56713.65$  total area (old version), then, using the  $\Sigma Sq$  curve, we find  $N_L= 5$  (five “long” and four “short” cycles) and then (Fig. 2b)  $\Sigma T(N_L=5) = 1180$  months, which is 2.5 years shorter than the tabular duration of the 1–9 cycles. In the new version,  $\Sigma Sq_0=95482.44$ ,  $N_L=4$  and  $\Sigma T(N_L= 4) = 1169$  months, which is 3.3 years shorter than the tabular duration.

V-3. Let us estimate the group parameters based on  $N_L= 6$ , i.e. general characteristics of its structure. For the old version, we get  $\Sigma T(N_L= 6) = 1194$ ,  $\Sigma Sq(N_L= 6) = 49807.86$ ,  $w \sim 41.68$ . For the new version,  $\Sigma T(N_L= 6) = 1194$ ,  $\Sigma Sq(N_L= 6)=88771.25$ ,  $w \sim 74.29$ . Again, we get an underestimated activity level ( $w < w_0 \sim 46.91/78.98$ ) with the group duration reduced by 15 months.

Moreover, it was impossible to use the total length of growth branches in this approach because of its significant value in the group of restored cycles, which was noted above and corresponds to the  $T_m$  values in the Table.

### Scenarios for the 1–9 cycle group G0

When projecting the “rules” obtained for the G1–G6 groups onto the G0 group, we see a mismatch between its duration and “energy”. The estimates obtained indicate the need to change, first of all, the Wrest series structure, i.e. correlation of “long” and “short” cycles (correction of the duration of individual cycles is also possible). Increasing proportion of “short” cycles leads definitely to the growth of average activity level. Several options for such a correction are given below.

Let us set the average  $\Sigma T$  value, being equal to 1159.0 months (Fig. 2a), by the G1–G6 groups for a weighted estimate of the G0 group length, which is very close to the estimate after the period of the basic harmonic of  $129 \cdot 9 = 1161$  months. We have a scenario of three “long” and six “short” cycles ( $N_L= 3$ ) with  $\Sigma Sq(N_L= 3) = 66975.50/100762.50$  (old / new version) and  $w \sim 57.8 / 86.9$ , which exceeds  $w_0$ . For close variants with  $N_L= 2$  and  $N_L= 4$  we have:

--  $N_L= 2$ ,  $\Sigma T(N_L= 2) = 1137.50$ ,  $\Sigma Sq(N_L= 2) = 72519.52 / 107177.02$ ,  $w \sim 63.8 / 94.2$ ;

--  $N_L= 4$ ,  $\Sigma T(N_L= 4) = 1170.50$ ,  $\Sigma Sq(N_L= 4) = 61209.02 / 95556.70$ ,  $w \sim 52.3 / 81.6$ .

Let us note that the requirements for the G0 group correction are more stringent in the new version of Wolf numbers. An overview of the options considered in the coordinates ( $\Sigma T$ ,  $\Sigma Sq$ ) for the old version of the G0 group is presented in Fig. 4 with clear separation of “weighted estimates” ( $N_L=2-4$ ) and “agreed parameters” (V-1 – V-3) areas.

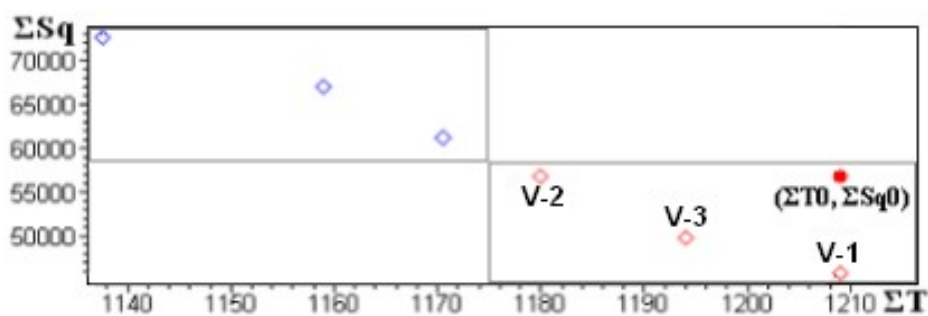


Fig. 4. Overview of scenarios for Group G0 (old version)

### Conclusions

When comparing extended fragments, the local data residual plays a smaller role and more balanced estimates are obtained. Classification of cycles by duration allows us to describe the relationship between the parameters of reliable cycles and to show the inconsistency of these parameters for the 1–9 cycle group. A more realistic scenario for the Wrest series corresponds to a simultaneous reduction in the length of the series by 4–5 years and an increase in the “average”  $W$  by 1.25 times.

These conclusions are consistent with the result of paper [2], where Wolf numbers and temperature series of several European centres are compared. It was noted in the work that, around 1925 and later, a tendency toward synchronization of temperature characteristics is visible, which

reflects the growth and increased influence of solar activity. A similar synchronization of temperature characteristics was also noted before 1850, that is, the necessary “increase” in solar activity in this interval (by analogy with the period after 1925) was superimposed on the Dalton minimum.

In conclusion, we note that a number of authors express a critical attitude to the restored series in the works of the Solar-Terrestrial Communications, Weather and Climate Symposium (1978) [4]. An attempt to balance the temporal characteristics of the W series cycles due to the “lost” cycle was undertaken in [5]. We also note the paper [6], analysing the series fractal properties for the annual ring width of eleven sequoias. The authors identified time intervals coinciding with the Speper and Maunder minima, without the Dalton minimum manifestation.

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