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DEDUCTION OF IONOSPHERIC PARAMETERS MEASURED ON BOARD OF "ICB-1300"

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Abstract

This work contains some examples of data, measured on board of the satellite 'ICB-1300', launched in August 1981, and could be useful to estimate the accuracy and reliability of the obtained physical parameters of the ionospheric plasma. Some methods for retrieving information are presented. Special attention was paid to the models and mathematical methods, applied to detect parameters measured by the complex of instruments for diagnostic of cold plasma, as well as a comparison between different approaches to the problem.

Introduction

Satellite 'ICB-1300' was launched in 7th of August 1981, and due to the reasons that will not be discussed here, significant amounts of the received data remains unused. Since interest in this data still exist, it may be useful to present some of the mathematical techniques, developed at the very beginning and continuously improved, applied to retrieve physical parameters from measured data. Moreover, most of the papers, concerning deductions of plasma parameters, do not consider mathematical and computational difficulties (exception for Moss and Hyman (1968), Patterson (1969)). Note that the purpose of the paper is not to suggest decisions about which method should be used for a particular problem, since most of them were tested according to their ability to handle uncertainties in the instruments output.

Mathematical background

The general form of the problem of processing experimental data can be presented as operator equation

$$Az = u\,, \tag{1}$$

where *A* is an operator, expressing, in general, a known relationship between *z* and *u*.
The type of measurement, performed by the instrument, leads to two different problems: direct and inverse. The direct one, i.e. "*z* is known, *u* is to be estimated" according to (1), correspond to the measurements where the physical parameter could be received by a single measured value z from the set of values encountered along the trajectory.
If the physical parameter could not be measured directly, but a function characterizing that parameter can be, the problem is known as inverse. Experiments, aimed to detect ionospheric plasma parameters: cylindrical Langmuir probe (CLP), spherical ion probe (SIP) and retarding potential analyzer (RPA), generate tasks of this type. Such problems usually are not well posed (or ill-posed) in the sense of Hadamard, i.e. small errors in the right part of Eq.(1) could produce big ones in the solution, which, as a rule, reflects the ambiguity of the solutions. For those experiments at least three sources of errors are known: errors due to the experimental character of data – usually having a normal distribution, errors due to the length of the telemetry word (9-bits) – uniformly distributed,

and those of the difference between theoretic form of *A* and the one, realized by the instrument. Note that the instruments mentioned above perform an operation of type integration of the formula of Maxwell distribution (by means of voltage), so the problem of deducing plasma parameters from current-voltage relationship is almost identical to the problem of numerical differentiation, ill-posed when metrics of *C* or L_2 is used as a measure of distance.

In the process of creating a software system to retrieve physical parameters, knowing that the problem is ill-posed with complex error's distribution, two different approaches were implemented: robust estimates in $Lp, 1 \le p \le 2$ and minimizing the error in space of Sobolev's W_2^1, and are to be described briefly below.

Robust estimates

Applying ideas of Huber (1964) to the linear regression model

$$Y = X\beta + \varepsilon$$

where *Y* – the current-voltage characteristic dimensioned (N), *X* – the construction matrix *dim* (*NxP)*, *β* – the solution vector *dim* (P), *ε* – random vector *dim* (N), an iteration process

$$\beta^{(k)} = \left(X^T W X\right)^{-1} X^T W Y \,,\ W \equiv diag(w_1, w_2, ..., w_n)\,,$$

with $$w_i = \frac{\psi\left[\left(y_i - \sum_{j=1}^{p} x_{i,j}\beta_j^{(k-1)}\right)\frac{1}{s}\right]}{\left(y_i - \sum_{j=1}^{p} x_{i,j}\beta_j^{(k-1)}\right)\frac{1}{s}}\,,\ i = 1,...,n;$$ $\psi[.]$ - *Huber's* function,

should be continued until reasonable convergence is reached. Initial estimate for *β* could be obtained following Patterson (1969), where linear system should be solved by procedures as NNLS (Lawson, Hanson, 1974) or by means of *Lp* metrics (Ekblom, Hakan, 1973), and minimization could be performed by the method of Powell (1989). Here the result of such technique is illustrated (Fig.1) on data from CLP (electron temperature and density) and SIP (ion density). As an approximation of the left part of current-voltage of Langmuir probe the model of Moskalenko (1975) is used,

$$u(\varphi) = I_e(\varphi) + \sum_{i=1}^{n} I_i(\varphi),$$

$$I_e(\varphi) = \begin{cases} eSN_e\left(\frac{\chi T_e}{2\pi m_e}\right)^{\frac{1}{2}} \exp\left(\frac{e(\varphi+\varphi_s)}{\chi T_e}\right), \varphi+\varphi_s \le 0 \\ eSN_e\left(\frac{\chi T_e}{2\pi m_e}\right)^{\frac{1}{2}}\left(\frac{2}{\sqrt{\pi}}\sqrt{\frac{e(\varphi+\varphi_s)}{\chi T_e}} + \exp\left(\frac{e(\varphi+\varphi_s)}{\chi T_e}\right)\left(1 - erf\left(\sqrt{\frac{e(\varphi+\varphi_s)}{\chi T_e}}\right)\right)\right), \varphi+\varphi_s > 0 \end{cases}$$

$$I_i(\varphi) = \begin{cases} eSN_i\frac{1}{\pi}\sqrt{w^2 - \frac{2e(\varphi+\varphi_s)}{m_i}}, \varphi+\varphi_s \le 0 \\ eSN_i\frac{1}{\pi}\sqrt{w^2 - \frac{2e(\varphi+\varphi_s)}{m_i}}\,\frac{1}{2}\left(1 + erf\left(\frac{w}{w_i} - \sqrt{\frac{e(\varphi+\varphi_s)}{\chi T_i}}\right)\right), e(\varphi+\varphi_s) \le \frac{m_i w^2}{2} \\ 0, e(\varphi+\varphi_s) > \frac{m_i w^2}{2} \end{cases}$$

which behave well when the ion saturation region is short, a common situation with the CLP on board, and for the electron saturation region (the right part of the curve), model, described in (Bankov et al. 1983).

Minimizing the error in Sobolev's space W_2^1

The metric in W_2^1 is defined by

$$\rho_W(z_1, z_2) = \left[\int_a^b \sum_{r=0}^{1} w_r(x) \left(\frac{d^r z}{dx^r} \right)^2 dx \right]^{\frac{1}{2}}, z = z_1 - z_2, w_r(x) \geq 0, w_1(x) > 0 .$$

As far as spherical ion probe was supposed to measure current-voltage characteristics together with their derivatives, the idea to solve the problem in W_2^1 was more than obvious. However, the measurements of the derivatives failed, but minimizing the error according to this metric works well even if a computed derivative in spite of measured one is used (Bankov N. 1987). Note that the use of the derivative itself (Wrenn 1969), or the position of certain point (Moss and Hyman 1968), is not unusual for such instruments. Note also, that the use of this metric is natural according to the method of regularization of Tihonov (1974). Thus, determining the derivative is important part of the proposed procedure. Here we will concentrate on a part of this problem.
The n-th derivative z(x) of the function u(x) solves integral equation of the first kind

$$\int_0^x \frac{(x-s)^{n-1}}{(n-1)!} z(s) ds = u(x) . \qquad (2)$$

As was noted above, this problem is ill-posed, and in order to find regular solution $z_\alpha(s)$, the regularization theory leads to the problem of minimizing

$$M^\alpha[z,u] = \int_c^d \left[\int_a^b K(x,s) z(s) ds - u(x) \right]^2 dx + \alpha \int_a^b \left[w_0(s) z^2(s) + w_1(s) \left(\frac{dz}{ds} \right)^2 \right] ds , \qquad (3)$$

where $a \leq s \leq b$, $c \leq x \leq d$, $[a,b] \in [c,d]$, $u(x) \in L_2[c,d]$, and *K(x,s)* denotes kernel in Eq(2). Usually the parameter of regularization, α, is determined from the condition $\rho_{L_2}(Az_\alpha, u) = \delta$, where δ should be an estimation of the error of *u* - right part of Eq.(1), which is probably possible to be measured for some ground-based experiments, but difficult to be estimated on board of a satellite. Here other criteria will be introduced, which work well, at least for monotonous functions. Consider the function

$$\beta(\alpha_k) = \frac{\rho_{L_2}\left[C.\mu\left(z^{(n)}\right), |Az - u_m|\right]}{\rho_{L_2}\left[0, |Az - u_m|\right]},$$

where $\alpha_k = const.\alpha_{k-1}$, $z^{(n)}$ - *n*-th derivative,($n \leq 2$), u_m denotes measured current-voltage characteristic, μ(.) – modulus of non-monotonicity (Sendov 1979),

$$\mu(f_i) = |f_i - f_{i-1}| + |f_i - f_{i+1}| - |f_{i+1} - f_{i-1}| , \; f_i = f(x_i \pm h).$$

The values for *C*, *const* and *n* should be evaluated by computer simulation, using analytical formula for current-voltage, noised with generator of random digits. An advantage of the function *β(αk)* is that it has an easy to detect minimum, and when *C*, *const* and *n* are properly estimated, it marks the value of α to be used in Eq.(3). A plot of current-voltage characteristic from spherical ion trap and its first order derivative (Fig.(2)-a). and b). correspondingly), obtained by this method, together with a plot of *β(αk)* (Fig.(2) - c) is present.

Note that this criterion could be applied for a smoothing spline function, or in a case when Eq.(3) has to be solved on a set of monotonous or monotonous-convex basis. However, we shall note that it does not work well when the level of noise for u_m is comparatively small (less than 1%).

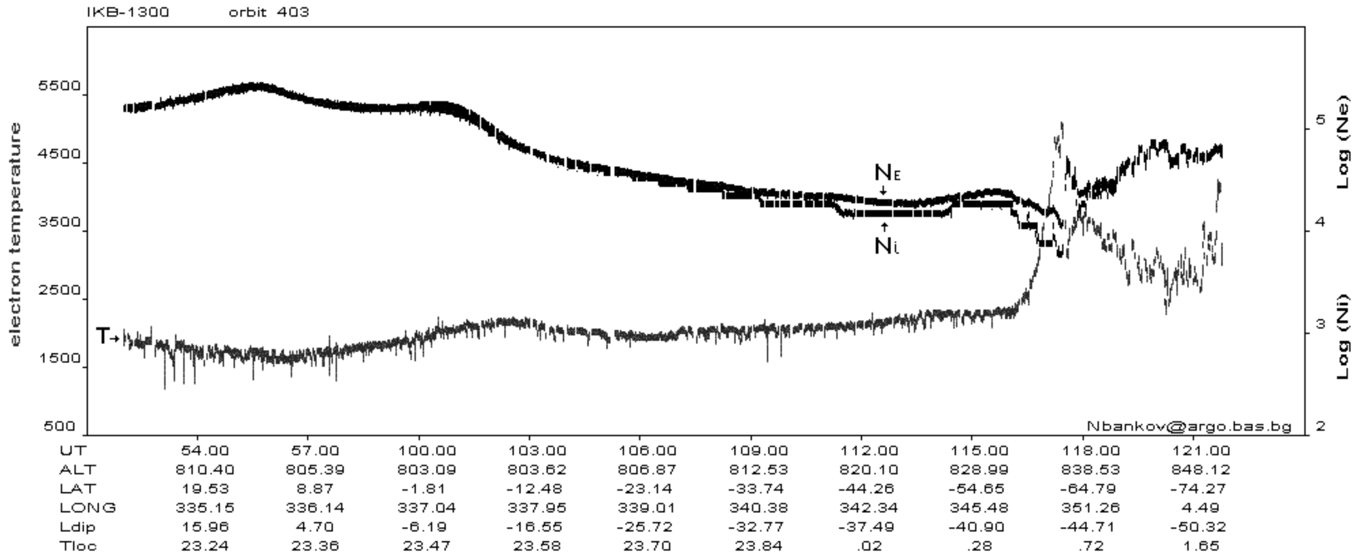

Fig.1

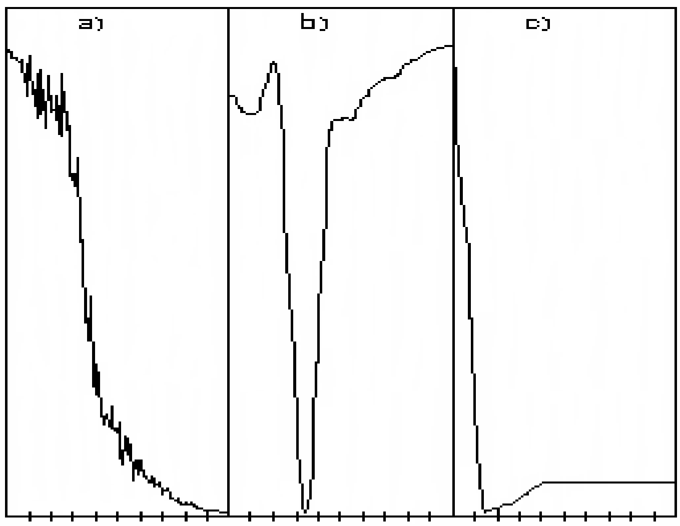

Fig.2

Acknowledgments

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