S E N S 2 0 0 9
Fifth Scientific Conference with International Participation
SPACE, ECOLOGY, NANOTECHNOLOGY, SAFETY
2–4 November 2009, Sofia, Bulgaria

MODELS OF VORTICES DURING THE ACCRETION PROCESSES IN CLOSE BINARY STAR SYSTEMS

Daniela Boneva, Lachezar Filipov

Space Research Institute - Bulgarian Academy of Sciences
e-mail: danvasan@space.bas.bg; lfilipov@space.bas.bg

Keywords: *accretion discs, binary stars, structures, spiral wave*

Abstract. *The examinations in the paper below are related to the problem of structures in the accretion flow around close binary stars. The main reason for studying vorticity is the assumption of many authors, which points to the basic role of these formations in angular momentum transportation through the accretion discs. The current survey presents results, which have been obtained applying different methods to the base features of accretion flow around close binary stars. Theoretical methods have been used in conjunction with numerical simulations that lead to obtaining the next graphical models of accretion processes. These models reveal: initially unstable behaviour of the flow; transitional states; spiral structure formation and spiral solutions of the ruling equations; vorticity undulations near the contact region of the two object's matter flows.*

Introduction

In astrophysics, examinations of accretion disc's structure, as a global formation or as locally patterns along accretion flow, are still actual. Most investigated of them are vortex formations and spiral configurations. Vortices may play at least two important roles regarding accretion disc dynamics. First, they could lead to an efficient angular momentum transport process in regions in which the magneto-rotational instability [3] doesn't operate.

Using global 2D hydrodynamical simulation Li [10] conclude that Rossby wave instability is an efficient purely hydrodynamic mechanism for angular momentum transport in thin disc. He found that individual vortices transport angular momentum outward. Lovelace [11] found such instability give rise to Rossby vortices in the non-linear limit and the persistence of vortices would be crucial for the hydrodynamic transport of angular momentum in accretion discs. It is also suggested from authors [13] that some instabilities or turbulizations in the viscous flow are responsible for the carrying out of angular momentum.

Second, they are a very efficient way to accelerate the planetesimal formation process in protoplanetary discs [6]. However, the stability of these vortices when small 3D disturbances are imposed is largely unknown.

In the astrophysics community, this issue has been investigated mainly numerically. Shen et al. [14] examined the formation of 2D vortices starting from 2D turbulence in fully compressible simulations. According to their results, a small 3D noise added to their initially 2D configuration destroys the coherent vortices in a few orbits, relaxing the flow to its laminar state. Barranco & Marcus [4] also computed the evolution of 3D vortices using an anelastic code incorporating vertical stratification. As Shen et al. [14], they found that midplane vortices were destroyed by 3D perturbations. However, they also showed that off-midplane vortices could survive for several hundreds of orbits, leading to the possibility of a stabilizing effect due to the stratification. Johnson and Gammie [7] have performed a series of runs with zero initial vorticity and perturbation wavelengths and give one possibility of what generates the initial vorticity. They note that a residual amount of vorticity can be generated from finite-amplitude compressive perturbations.

Currently, how we have seen, an explicit answer doesn't exist. The main aim of current survey is to present our study of several models, which support explanation of physical processes of vorticity formation. In a connection with these problems research methods are specified in the next sections of our article.

The first point is based on the bifurcation theory, which is a powerful tool for analyzing the nonlinear evolution of instability behavior in pattern forming systems, such as in the accretion disc structure [8]. In this paper we present the results of performing this computation in a neighborhood of a Turing bifurcation. Also, we apply the spatial-temporal behavior of Hoph bifurcation as the key mechanism for stability-instability activity. The second part is based on the appearing of structures as a consequently result of baroclinicity in the disc flow. To receiving of the results graphically we are applying appropriate numerical code.

2. Structures formation, as a result of applying the bifurcation analysis

Here we use a different mechanism to explain the instability operation and vorticity formations in the accretion discs. This part includes some basic bifurcation analysis, which we may apply, to detect these processes.

By definition, a bifurcation is any qualitative or topology reconstruction of the system, when the parameter of the system crosses its critical value. We initially denote the critical value of the control parameter of the system with λ_c, which is in fact a bifurcation point. When a given system passes through a bifurcation point, it may lose its stability. Then conditions for forming structures in these places grow up. The obtained results are of a global character, but they have an effect on the local behaviour. When we talk about bifurcation, we have to specify that there exist a different kind of its evidence. The essence of different kind of bifurcation is defined from the relevant amplitude equation [12].

We need bifurcation solutions, related to the spatial – temporal pattern formations. Therefore, we use the Hoph and Turing bifurcations and instabilities. Hoph instability results in spatially homogeneous temporal oscillations and its relation to Turing instability is of use for our current reasoning.

It is seen in figure (1a) these consequently transitions. The obtained results are of a global character, but they have an effect on the local behaviour. This means that in the temporal-spatial considerations of the systems, Turing and Hoph bifurcations are in action.

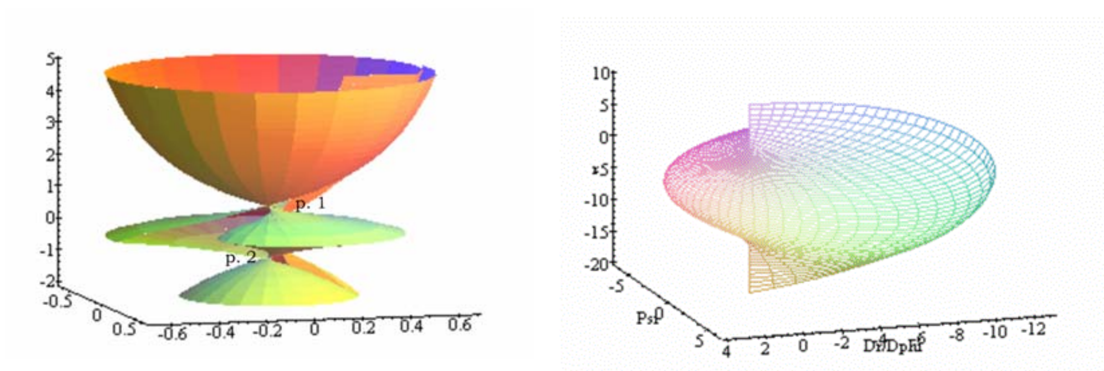


Fig. 1. a) Consequently transition through the bifurcation points 1 and 2. Increasing conditions for structure formation /left/. b) Spherical vertical structure, received by the existence of Turing instability. It is applied a 3D graphical simulations code on the equations ruling the conditions /right/.

The Turing instability is an expression of spatial model of the bifurcation area and they may cause the formation of structures, such as: spherical structures in 3D model (fig. 1b) or in a 2D consideration – the structures are limited to the appearing of spot and disturbances of laminar flow.

As a base equation, it is used the vortical transport equation and we referred to the conditions of Turing instability that follow this equation (see [1]). It is applied a numerical parameterization code, which is based on the Runge-Kutta method. For a better decipher of the results, a set-scale of [12,12,12] is more appropriate. In a result, a development of vortical spiral structures is obtained.

Hoph bifurcation is described by the amplitude equation and includes in its form periodic function. Garaud and Ogilvie find in their work [5] that the behaviour of the system depends on the Rayleigh discriminant of the rotating shear flow. The model predicts that Rayleigh unstable flows become turbulent at sufficiently large Reynolds number through a linear instability associated with a supercritical bifurcation. Here we start our studying of Hoph bifurcation using these criteria. To describe the Hoph bifurcation we represent the basic flow in dimensionless variables, satisfying the Navier-Stokes equations, so:

R - Reynolds number: $R = WL / u$; W - characteristic velocity scale; U_*- velocity field and P_*– pressure of the basic flow; L - characteristic length scale.

Lets introduce the quantities: $x = x_* / L, t = Vt_* / L, u = u_* / V, p = p_* / \rho V^2$; here $u_* p_*$ are the quantities at a given point x_* at time t_*.

The Navier-Stokes equation now is in the form, with velocity field $U(x,R)$ and pressure field $P(x,R)$:

(1) $$U.\nabla U = -\nabla P + R^{-1} \Delta U;$$

In general U, P depend on R and there may be more then one steady solution for the same value of R and the same boundary conditions.

Our next step is to construct model amplitude equations of these ordinary differential systems.

The sequence of instabilities and changes of flow regime, as the Reynolds number increases, may be scrutinized as a physical idea by very simple model problems of bifurcation. These models represent asymptotically and local properties of many stability, instabilities and bifurcations of solutions of the Navier-Stokes equations, governing the flow of fluid.

We take the next model equations, which are constructed on the base of amplitude equations, used from Nicolis [12]:

(2a) $$\frac{dx}{dt} = -l + (a - x^2 - l^2)x;$$

(2b) $$\frac{dl}{dt} = x + (a - x^2 - l^2)l;$$

where $a = k(R - R_c), k > 0$;

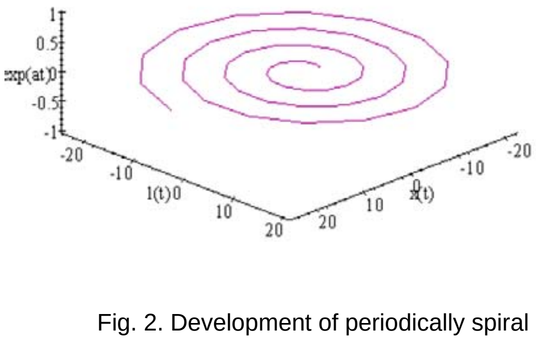
It is seen that the only steady solution of this system is its null: $x = l = 0$. Also, we suppose that $x, l \propto e^{st}$, as $s = a \pm i$. Now, if A is complex amplitude constant then we received:

(3a) $$x(t) = \frac{1}{2}(Ae^{it} + A^* e^{-it})e^{at};$$

(3b) $$l(t) = -\frac{1}{2}i(Ae^{it} - A^* e^{-it})e^{at};$$

As we use these Eqs. (3a), (3b), we may define one simple condition for stability: when $R < R_c$ there is stability with exponential decay and at $R > R_c$ there is instability. Here R_c is our critical point value.

It is shown at the figures below, the case $R > R_c$, the periodically unstable behaviour of the system through the Hoph bifurcation point.



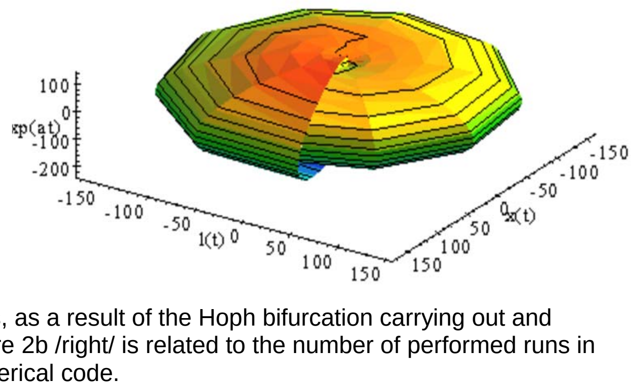


Fig. 2. Development of periodically spiral structures, as a result of the Hoph bifurcation carrying out and solutions. The difference between figure 2a /left/ and figure 2b /right/ is related to the number of performed runs in applied numerical code.

It is seen from the Fig. 2a and Fig. 2b that the solution has a spiral form and represents the results due to the bifurcation transition in the hydrodynamical flow, such as the accretion discs are. This fact confirms our initially predictions for the results in the form of structures. In this section we used the ordinary differential equations to simplify the model and we proceed from the conception of introduced form of presented bifurcations. In our future studies, for the decision of the Hoph bifurcation problem, we plan to use partial differential equations, because they contain more complete information of the fluid dynamics.

Development of vortices in accretion flow.

In this section, we are taking into account the presence of baroclinic instability in the discs. This theme is mainly considered by Klahr & Bodenheimer [9] and we have concerned to it in detail in [2]. Here, the last results are presented, as a sequel of previous research.

Further development of the baroclinic instability turns it into the source of vorticity formations in the discs. At the Fig. 3 we present our simulation of this kind of pattern growth in R, φ plane of the disc zone. We have performed a series of runs applying numerical code, with zero initial vorticity, but with initial present of turbulization value different from zero. We consider the gas-flow in the disc to be compressible and viscous.

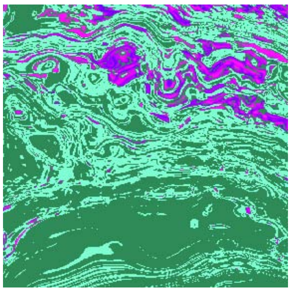


Fig.3a

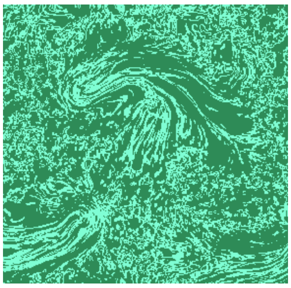


Fig.3b

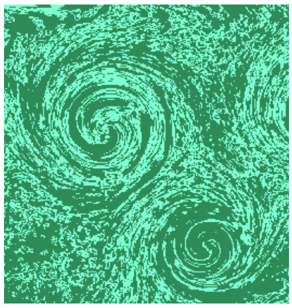


Fig.3c

Fig. 3. Simulation of the vortices formation as a result of the baroclinic instability. It is seen in the figures (3a, 3b, 3c) the consequently development of vortices: from weakly turbulization of the layers to the stage, when the separated structures become visible in the flow. The resolution is 1024 x 768 of the grid measures.

In accordance to the conditions, it is required misalignment of pressure gradient and density gradient wherever azimuthal density gradient appears. This misalignment acts as a source term for the generation of vorticity in the region of outer edge of the disc with strong density gradient.

The pictures' frames are visualizing a single vortex, which is a part of covered range of about 7.687 $\times 10^{-6}$ AU to 6.68 $\times 10^{-5}$ AU. The development of such vortex, pass through a few stages. First, it is observed a distortion of laminarity of the flow (fig. 3a) and weekly undulation of the layers (fig.3b). We confine to the stage of the vortex evolution in some steady period of their development, when they are "ready" for the angular momentum transportation (Fig. 3c).

Such vortices may propagate throughout the disc, according to the baroclinicity global character, but they are local, temporal formations. How long these vortices will live and what would be destroyed them is another unanswered questions and undecided problems. The recent numerical results of [4] indicate that vortices near the disc midplane are quickly destroyed, whereas vortices survive if they are on scale heights away from midplane.

Conclusion

One of the most vexing problems in astrophysics is how mass and angular momentum are transported in accretion disks. It is considered from many authors that the vortices have a vital role in such transfer and this is the main reason to investigate and create models of vortex development. The appearing of vortices in the disc's flow is in a conjunction with some wave processes and spiral structure formation, which is in an accordance of wave theory. Here are the base results of our studying of vortical models in accretion disc's flow:

- Using bifurcation analysis it is revealed the transitional states through the bifurcation critical point. The solutions show the model that describes initially development of spirals, related to Turing bifurcation problem. After operation of Hoph bifurcation, the results show periodically spiral wave solutions to be appeared in examined accretion flow area.

• At certain conditions and after applying an appropriate numerical model, the result shows an existence of two-dimensional vortical structures configurations. These vortices are considered as a resulting effect of foregoing baroclinic instability.

This type of vortices often observed in Two-dimensional simulations of discs in the last ten years. Vortices can be generated by Two-dimensional instability as the Rosby wave instability or baroclinic instability, which have been investigated in recent years. Tidal waves from donor star, which usually cause a development of spiral density star, could be also responsible for vorticity in accretion area. But the models of such effects are the topic of another paper.

References:

1. A n d r e e v a/B o n e v a/, D. V., F i l i p o v, L. G., D i m i t r o v a, M. M., 2002. Publ. Astr. Obs. Belgrade, **73**, 67-71
2. A n d r e e v a/B o n e v a/, D.V., 2005, Conf. Paper Coll., Space Ecology Safety, Varna, Bulgaria, 25, b. I, http://www.space.bas.bg/astro/ses2005/a3.pdf
3. B a l b u s, S. A. & H a w l e y, J. F. 1998, Reviews of Modern Physics, 70, 1
4. B a r r a n c o J. A., M a r c u s P. S., 2005, ApJ, 623, 1157-1170
5. G a r a u d, P., O g i l v i e, G., 2005, J. Fluid Mech, **530**, 145-176
6. J o h a n s e n A., A n d e r s e n A., B r a n d e r b u r g A., 2004, A&A, 417, 361
7. J o h n s o n B. M., G a m m i e C. F., 2005, ApJ, 635, 149-156
8. J u d d, S L., S i l b e r, M., 2000. Simple and Superlattice Turring Patterns in Reaction-Difusion Systems: Bifurcation, Bistability and Parameter Collapse, Physica D, v.**136**, 45-65
9. K l a h r H., B o d e n h e i m e r P., 2003, ApJ, **582**, 869-892
10. L i H., C o l g a t e S. A., W e n d r o f f B., L i s k a R., 2001, ApJ, **551**, 874
11. L o v e l a c e R.V.E., L i H., C o l g a t e S.A., N e l s o n A.F., 1999, ApJ, **513**, 805-810
12. N i c o l i s, G., 1995. Introduction to nonlinear Science, Cambridge University Press of the binary IP Pegasi', M.N.R.A.S. 290, L28.
13. P e t e r s e n M.R., S t e w a r d G.R., J u l i e n K., 2007, Ap J., 658:1252-1263
14. S h e n, Y., S t o n e, J. M., & G a r d i n e r, T. A. 2006, ApJ, 653, 513