

## **MODEL STUDY OF ELECTRIC FIELDS GENERATED BY THUNDERSTORMS IN MAGNETICALLY CONJUGATED REGION**

**Peter Tonev**

*Institute for Solar-Terrestrial Influences - Bulgarian Academy of Sciences*  
e-mail: ptonev@bas.bg

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**Abstract:** *Steady-state electric fields and post-lightning quasi-electrostatic fields (QESF) generated in different atmospheric regions above thunderstorms play an important role as drivers of a diversity of processes and events related to electron heating, such as, the newly discovered transient luminous events. Numerous recent investigations of these electric fields and events allowed better understanding of the electrical coupling of atmospheric regions and showed their importance. However, the problem of whether the electric fields generated by a thunderstorm can penetrate into its magnetically conjugated region and what effects they could cause there has not been studied sufficiently. Some observations in the mesosphere, such as the large (1 V/m and more) QESF, which have been measured occasionally for decades at mesospheric altitudes, could be explained as such effects, as indicated by some authors. Furthermore, red sprites have been observed, with no causative thunderstorm in a surrounding of thousands of km - these are considered by some authors to be the result of lightning in the magnetically conjugated region. We propose a 2D numerical model for the generation of QESF due to a lightning discharge in the conjugated region. The model is based on the Maxwell's equations for quasi-static conditions, which take place in the mesosphere and at higher altitudes in both hemispheres. Our results show that in the mesosphere these electric fields can be as large as few tenths of V/m. Because of their large horizontal scale (thousands of km), the observed larger electric fields may be the result of several lightning discharges occurring almost simultaneously.*

## **МОДЕЛНО ИЗСЛЕДВАНЕ НА ЕЛЕКТРИЧЕСКИ ПОЛЕТА ГЕНЕРИРАНИ ОТ ГРЪМОТЕВИЧНИ БУРИ В МАГНИТОСПРЕГНАТАТА ОБЛАСТ**

**Петър Тонев**

*Институт по слънчево-земни въздействия - Българска академия на науките*  
e-mail: ptonev@bas.bg

**Ключови думи:** *глобална атмосферна електрическа верига, квазиелектростатични полета, гръмотевична буря, мълниев разряд облак-земя, магнитоспрегнатата област, числено моделиране*

**Резюме:** *Стационарните електрически полета и квазиелектростатичните полета след мълния (КЕСП) генерирани в различни атмосферни области над гръмотевични бури играят важна роля като агент за осъществяване на разнообразни процеси и явления свързани с електронно нагряване. Такива са, например, новооткритите преходни събития на светене в средната атмосфера. Многобройните съвременни изследвания на тези електрически полета и събития позволяват по-добро обяснение на електрическите взаимодействия между атмосферните области и показват важно значение на тези взаимодействия. Но проблемът дали електрическите полета от гръмотевична буря могат да проникват в магнитоспрегнатата ѝ област (МСО) и да причиняват ефекти там е недостатъчно изследван. Някои наблюдения в мезосферата биха могли да се обяснят като такива ефекти, например, големите (1 V/m и повече) КЕСП измервани понякога в мезосферата. Също, наблюдавани са спрайтове без поражаваща мълния, обяснени от някои автори като резултат от мълния в МСО. Ние предлагаме 2D числен модел за изследване на КЕСП причинени от мълния в МСО. Моделът е базиран на уравненията на Максвел при квазистатични условия, които имат място в йоносферата и по-нагоре при мълния. Резултатите ни показват, че в мезосферата тези електрически полета достигат десети от V/m. Тъй като СА с голям хоризонтален мащаб (хиляди км), наблюдаваните по-големи електрически полета могат да са резултат от няколко почти едновременни мълнии.*

## Origin of the mesospheric electric fields and model basis

A strong lightning discharge generates large quasi-electrostatic fields (QESF) above the thunderstorm, whose relaxation is much longer than the relaxation of a free charge. These features are demonstrated by the results in Fig.2 [5] obtained by night-time conductivity profiles shown in Fig.1. The QESF and the related Maxwell currents are widely investigated (theoretically and experimentally) as possible drivers of red sprites and halos in the mesosphere [1, 6]. However, almost nothing is known about the role of these electric fields at higher altitudes. Although much smaller and shorter living, these fields can penetrate above the lower ionosphere, as shown experimentally in [3]. If this is the case, the electric fields at the magnetospheric basis could induce similar fields in the magneto-conjugated region, where they could generate measurable quasi-electrostatic fields in the lower atmospheric regions estimated from the parameters of the electric field pattern at 150 km in the conjugated region. A similar mechanism is theoretically discussed in [2]. Under the assumption that the QESF are potential in the strato/mesosphere and above [4], they can be computed in both regions as a solution of the continuity equation for the density of the Maxwell's current  $\mathbf{j}$ :

$$(1) \quad \nabla \cdot \mathbf{j} = 0, \quad \text{where} \quad \mathbf{j} = \mathbf{j}_C + \mathbf{j}_D.$$

Here  $\mathbf{j}_C$  is the conduction current,  $\mathbf{j}_C = [\sigma]\mathbf{E} = [\sigma]\nabla U$ , where  $[\sigma]$  is the conductivity tensor (a scalar below 70 km),  $\mathbf{E}$  is the electric field, and  $U$  is its potential;  $\mathbf{j}_D$  is the displacement current,  $\mathbf{j}_D = \varepsilon_0 d\mathbf{E}/dt$ ,  $\varepsilon_0$  is the dielectric constant. To estimate the QESF generated by a lightning discharge in the magnetically conjugated hemisphere, we solve Eq.(1) both in the region (**A**) of the lightning discharge and in its conjugated region (**B**) (Fig.1). The following assumptions are used: **a**) axial symmetry takes place in each region; and **b**) the electric fields at 150 km are transported without changes from the source region to its magnetically conjugated region. Our model neglects the Earth's curvature.

## Simulation of electric field temporal-spatial distribution

We study the temporal and spatial distribution of the quasi-electrostatic field  $\mathbf{E}$  generated by a cloud-to-ground (CG) lightning discharge, which acts in region (**A**), and their induction in the conjugated region (**B**) through the magnetosphere. These regions are schematically presented in Fig.3 as large horizontal scale (thousands of kilometers) circle regions which contact to each other at the magnetic equator. The source lightning discharge is represented in the model by the altitude  $Z_C$  and magnitude  $Q_0$  of the initial charge (at  $t=0$ ), and by a function of the charge decay  $Q(t)$  by  $t>0$ . In order to obtain the distribution of  $\mathbf{E}$  in region (**A**), as well as in its magnetically conjugated region (**B**), we solve an equation for  $U$  derived from Eq.(1) in cylindrical coordinates  $(r, \varphi, z)$  ( $z$  is a vertical axis through the charge removed and  $z=0$  is the ground level). This equation is:

$$(2) \quad \left( \sigma_P \frac{\partial^2 U}{\partial r^2} + \frac{\sigma_P}{r} \frac{\partial U}{\partial r} + \sigma_0 \frac{\partial^2 U}{\partial z^2} + \frac{\partial \sigma_0}{\partial z} \frac{\partial U}{\partial z} \right) + \varepsilon_0 \frac{\partial}{\partial t} \left( \frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial z^2} \right) = 0$$

Here  $\sigma_0$  and  $\sigma_P$  are the field-aligned and the Pedersen conductivities (the Hall conductivity is neglected in order to preserve the assumption for axial symmetry, which is required in our 2D model). In region (**A**) the coordinate  $z$  represents the altitude in the region of isotropic conductivity below 70 km, and an 'equivalent altitude' determined with respect to the magnetic field inclination (above 80 km  $z$  is the distance along field lines). In region (**B**)  $z = 300 - z'$ , where  $z'$  is the 'equivalent altitude'. The initial and boundary conditions are:

*Initial condition:*

The steady-state electric field in region (**A**) at the discharge beginning  $t=0$  formed by a constant charge  $Q_0$ , is determined analytically [5, 6] by using a modified boundary condition:  $U=0$  at  $z=Z_{\max}$ .

*Boundary conditions:*

- (i)  $U = 0$  at  $z = 0$  and at  $z=Z_{\max}=150$  km;
- (ii)  $U = 0$  at  $r=r_{\max}$  (at lateral boundaries in **A** and **B**);
- (iii)  $\nabla \cdot \mathbf{E}(t, r=0, z=Z_C) = Q(t) / \varepsilon_0$  at any time  $t \geq 0$  (after lightning onset);
- (iv) the Maxwell current  $j_z$  through the boundary at 150 km is continuous.

Eq.(2) is solved numerically in two steps on a regular rectangular grid with  $n_r=15 \times n_z=40$  cells which represents both regions **A** and **B**, as shown schematically in Fig.4. On the first step Eq.(2) is solved in region (**A**), and on the second - in region (**B**) where a boundary condition for the vertical Maxwell current  $j_z(t)$  at  $z'=150$  km is supported by the first step. On each step a finite element method approximation in  $\mathbf{C}^1$  is used by spatial components. It yields a system of linear equations whose form depends on the comparison of the local relaxation time  $\tau_R$  and the current time  $t$ , and thus changes with the progress of time  $t$ . Equations which correspond altitudes with relaxation time  $\tau_R > t/10$  are linear

differential, else they are transformed to linear algebraic. The total system of equations is solved by the Runge-Kutta method of 4<sup>th</sup> order (applied to the differential equations) and with the Givens method (for the algebraic linear equations). An increasing time step is used, which is controlled by the relaxation time at the lowering boundary between the regions of non-negligible and of negligible displacement current.

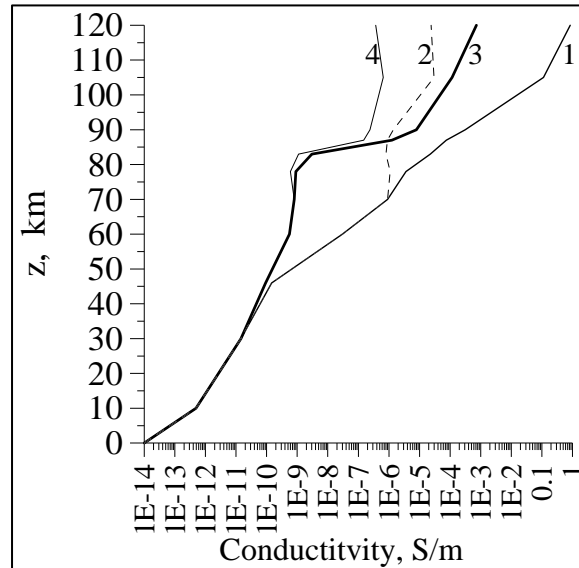


Fig.1. Conductivity profiles for daytime (curves 1 and 2) and nighttime (curves 3 and 4), adopted as a combination of different theoretical results and measurements. Lines 1 and 3 are for the field-aligned conductivity, and lines 2, 4 - for the Pedersen conductivity.

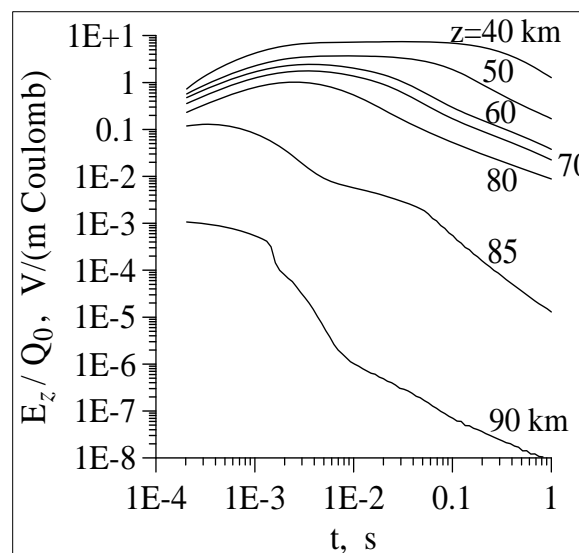


Fig. 2. Time dependence of the normalized vertical QESF  $|E_z/Q_0|$  at altitudes  $z=40 - 90$  km above a CG lightning discharge with  $Z_C=10$  km and by an exponential time decay of the initial charge  $Q_0$  in characteristic time  $\tau_L=1$  ms. Night-time conductivity profiles shown in Fig.1 are used (under assumption for isotropic conductivity).

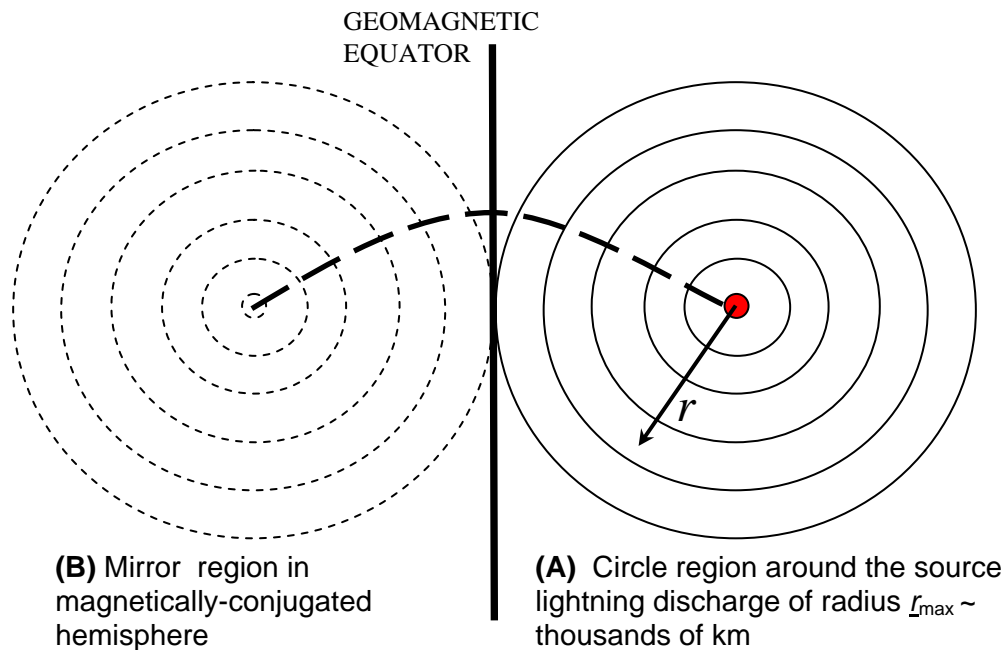


Fig. 3. Model domain: ● - location of lightning; ⋯ - magneto-conjugated image; — — — - geomagnetic field line;  $r$  - radial distance (up to 10000 km) from the lightning discharge (in region A) or from its image (in region B).

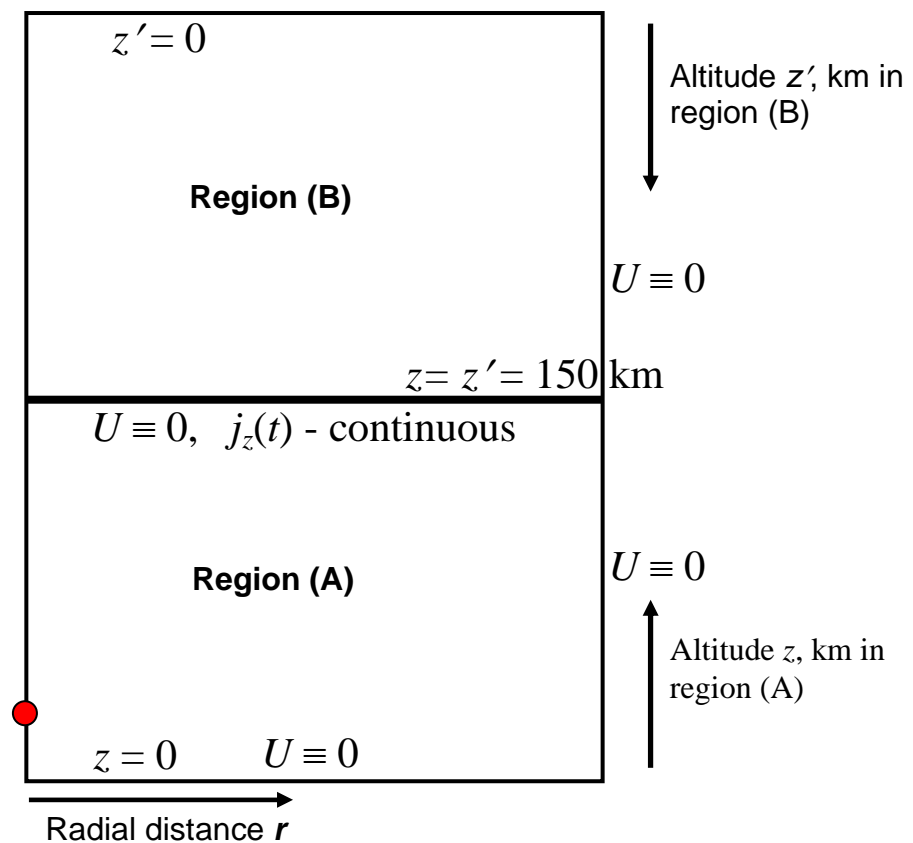


Fig. 4. 2D region used in numerical model.  $z, z'$  are altitudes in regions A and B, respectively. Thick line is a common boundary at altitude of 150 km for each region; ● - position of the charge removed.

## Results

Model results are obtained for the QESF generated by a positive cloud-to-ground lightning discharge in the magnetically conjugated region. The lightning discharge is realized from altitude  $Z_C=15$  km to the ground by an exponential decay of the removed charge  $Q(t) = Q_0 \exp(-t/\tau_L)$ , where  $\tau_L = 0.001$  s. The discharge occurs at distance  $r_{\max}=2000$  km from the magnetic equator. The results obtained are normalized to an initial charge  $Q_0=1$  C. Night-time conductivity profiles (the same in both regions **(A)** and **(B)**) adopted from [8, 9, 10] are used.

Fig.5 shows the temporal behavior of the quasi-electrostatic field  $\mathbf{E}$  in the magnetically conjugated region at different altitudes for  $r=0$ , where its vertical component  $E_z$  (and the electric field intensity) reaches a maximum. Our estimation for the electric field due to strongest lightning with a charge moment change, say,  $4500 \text{ C}\times\text{km}$  is that it will reach  $0.1 \text{ V/m}$  at  $70 \text{ km}$  and  $0.02 \text{ V/m}$  at  $60 \text{ km}$ . These values are relatively small; they can not explain the large mesospheric QESF measured occasionally [12], and do not support the suggestion that red sprites can be initiated by lightning discharges in the magnetoconjugated region [11]. However, they can be several times bigger if the electric field penetrates downward into an atmospheric layer with a much sharper conductivity decrease than typically. Such situation can take part, e.g. due to an enhancement of volcanic aerosols in thin layers in the strato/mesosphere, as predicted in [7], or by some other changes in the atmospheric content.

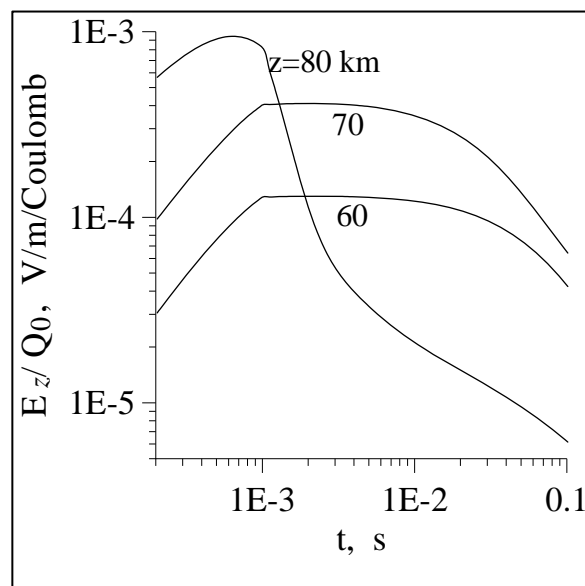


Fig.5. Time dependence of the normalized vertical QESF  $|E_z/Q_0|$  at altitudes  $z = 60, 70$  and  $80 \text{ km}$  generated due to a CG lightning discharge in the magnetically-conjugated region by  $Z_C=15 \text{ km}$  and by an exponential time decay of the initial charge  $Q_0$  in characteristic time  $\tau_L=1 \text{ ms}$ .

The horizontal scale of the QESF  $\mathbf{E}$  in the magnetically conjugated region **(B)** is studied. The results in the Table show the coefficient of the vertical electric field  $E_z$  decrease at a horizontal distance  $r$  compared with its value at  $r=0$ . We see that  $E_z$  decreases  $e$  times approximately at  $r \approx 1600, 1000$  and  $400 \text{ km}$  at altitudes  $z=60, 70$  and  $80 \text{ km}$ , respectively. Thus, the horizontal scale of the QESF in the mesosphere is much larger than above it. Because of that, the electric fields studied can results by many thunderstorm cells which occupy a region of dimension thousands of kilometers.

Table. Relative decrease of the vertical quasi-electrostatic field  $E_z$  induced due to CG lightning discharge in the magnetically-conjugated region with horizontal distance  $r$  from its maximum at  $r=0$ .

$r, \text{ km}$	133	267	667	1330
$z=60 \text{ km}$	0.95	0.89	0.77	0.48
$z=70 \text{ km}$	0.91	0.81	0.54	0.27
$z=80 \text{ km}$	0.85	0.73	0.24	0.11

## Conclusions

- The strongest lightning discharges cause in the magnetically conjugated region quasi-electrostatic fields of order of magnitude up to  $10^{-1}$  V/m at mesospheric altitudes (60-70 km);
- These quasi-static electric fields enlarge their horizontal scale to when penetrate into mesospheric altitudes from above.
- The mesospheric electric fields due to lightning in the magnetically conjugated region can be dramatically increased due to the following factors: *i*) QESF may result from many thunderstorms, due to large horizontal scale of these fields, especially at high geomagnetic latitudes; *ii*) QESF enhance at the upper boundary of a layer of significant local decrease of the conductivity due to enhancement of volcanic aerosols.
- QESF significantly change due to the highly variable conductivity profiles.

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