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EFFECT OF THE ROSSBY NUMBER VARIATIONS ON DYNAMICAL FORMATIONS IN ACCRETION DISC FLOW

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Abstract: *Using the known formulation and properties of Rossby number, we estimate its range of possible values and we study the relation to the turbulent scales of the accretion disc flow. In this paper, it is shown how the variable values of Rossby number are associated with two- or three dimensional flow motions. We consider the behavior of basic equations of accretion disc with the presence of Rossby number. Finally, as a connection with presented considerations, it is accented on the arising of vortex formations and their dynamics. We show the indirectly influences on the light curve form of one selected binary star. We perform calculations of Rossby number based on binary star systems models with accretion disc.*

I. Introduction

Fluid motion and hydrodynamic processes are one of the most important investigations in accretion disc dynamic. We consider that the flow in accretion disc is differentially rotating and in this survey we imply an additional term having influence on the instability processes. Such term is known as Rossby number named for Carl-Gustav Rossby, who first explained the large-scale motion in the atmosphere, using the terms of fluid mechanics.

In our study we use the next definition of Rossby number:

The dimensionless ratio of inertia force to Coriolis force which gives an indication of the importance of rotation on flow. It is given by:

$$R_0 = \frac{|v.\nabla v|}{2|\Omega \times v|} = \frac{v}{2\Omega L \sin\theta}$$

where V is the speed of fluid flow, Ω is the angular velocity or rotation, θ is the angle between the axis of rotation and the direction of fluid motion and L is the horizontal length-scale. The definition is taken from Blandford (2004).

This number plays a fundamental role to determine the behavior of large-scale astrophysical fluid dynamics. Large-scale flows are defined as those that are significantly influenced by the star's rotation and with sufficiently large *L* for Ro to be order one or less (e.g., flow with sufficiently small Rossby number are in geostrophyc balance).

Richard (2003) recently proposed a simple model describing the necessary conditions for self-sustained turbulence in differentially rotating flows, where it can be seen that the energy extraction is directly proportional to the shear present in the base flow (a classical result for shear flows). He also implies that there is a critical Rossby number, above which the energy extraction is sufficient to compensate for the stiffness introduced in the system by the mean rotation:

$$R_O = \frac{r \, \partial/\partial r \, \Omega_O}{2\Omega_O},$$

where *r* is the local radius and Ω_O is the mean rotation.

But, it has to be pointed (as Chagelishvili et al. (2003) also suggested) that the non-linear interactions (also referred to as turbulent diffusion) do not participate in the energy extraction, but only redistribute it and counteract the effects of rotation.

In this scenario, once the critical Rossby number has been reach (Richard 2003) and the critical amplitude is present, the flow can then undergo a transition from its laminar state to a state where non-linear shear turbulence is developed.

In the paper of (Morize et al. 2005) it is investigated the decay of initially three-dimensional homogeneous turbulence in a rotating frame. They found that during the decay, strong cyclonic coherent vortices emerge, while the Rossby number value decreases below the value of 2 ± 0.5.

Several simulations were conducted to examine the nature of balance in rotating stratified turbulence at different Rossby numbers (McKiver and Dritschel, 2006). They found a significant difference in the behavior of potential vorticity at low and higher Rossby number. The balance of vorticity decreases with increasing Rossby number and the structures which appear in the non-balanced field are mostly in small scale.

In the current survey we use the simplified formulation and expressions to examine the features of Rossby number and estimate its values in the case of astrophysical accretion flow. We bring to stay here in three cases of our considerations: A, B, C, presented in a next section.

II. Calculations and results

It is known that the rotation is able to confine larger scales in a two-dimensional state while efficient turbulent diffusion is achieved by three-dimensional motions. Typical geophysical rotating flows exhibit both two-dimensional (at large scales) and three-dimensional (at smaller scales) structures.

Results A

The Rossby number for the mean flow can be rewritten now as:

(1) $$R_0 = \frac{1}{2}\frac{r}{(\partial_r \ln\Omega)^{-1}}$$

where it appears as the ratio of local radius r over the characteristic length scale of the shear, Ω is the angular velocity again.

We may give the relation with a quantity more often referred to in astrophysics, the epicyclic frequency ω:

(2) $$\frac{\omega^2}{4\Omega^2} = (1 + R_0)$$

Using this formulation we estimated the range of critical values of Rossby number with the connection of quantities of frequency and angular velocity. We present the results graphically: Fig 1a and 1b.
We see from the figures that the values of Rossby number are low and it works at limited frequency range.

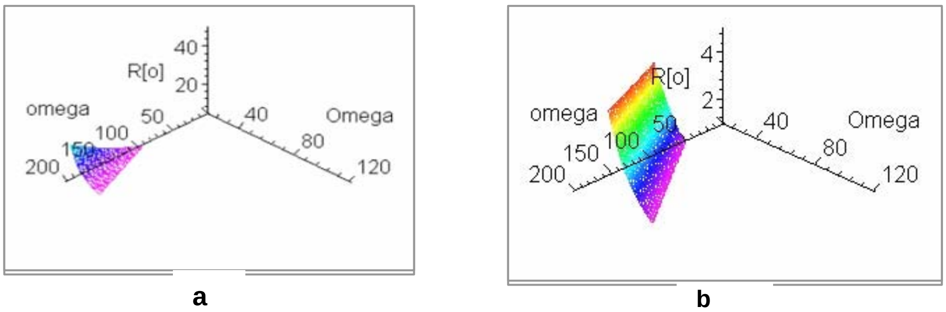


Fig. 1. *Graphic interpretation of simple dependence between ω, Ω and Ro. In the left panel /fig.1a/ we used small scale value for Ro calculations. In the right panel /fig.1b/, it is applied higher scale value. It is seen that the results are in a same range of values*

The Rossby number of a turbulent structure of characteristic length scale λ and rotating with mean flow velocity u can be approximated by $R_{O\lambda} \propto ru/2\lambda^2\Omega$ (known as "turbulent Rossby number"). The denominator of such expressed Rossby number is twice the rotation experienced by the turbulent structure, i.e. $2(\Omega \pm u/r) \cong 2\Omega$. The numerator is the derivative with respect to the radius of the turnover time, i.e. $r\partial_r\Omega + r\partial_r\Omega(u/\lambda) \cong r\partial_r\Omega + ru/\lambda^2 \cong ru/\lambda^2$ where we have used the relation:

(3) $$\frac{u}{\lambda} \infty r\partial_r\Omega$$

We can then write, from Eq.(3) and from the condition $\lambda << r$ that $R_{0\lambda} \infty \frac{r}{\lambda} R_0$.

Hence, the ratio of the characteristic turbulent length scale over the local radius writes as:

(4) $$\frac{\lambda}{r} \infty \frac{R_0}{R_{o\lambda}}$$

From the last relation, eq. (4), it follows that (for a given Rossby number) the turbulent Rossby number increases when going to smaller scales along the turbulent cascade. Baroud et al. (2003) have shown that low Rossby numbers are associated with two-dimensional turbulence whereas higher Rossby numbers are associated with three-dimensional turbulent structures. This result is consistent with the classic picture of a turbulent spectrum showing two-dimensional structures at large scales and three-dimensional ones at smaller scales.

Results B

To obtain the information for the effect of Rossby number we imply its expression in the main equations related to accretion disc dynamic. We present the results as follows:

We interpolate the dimensionless Rossby number into the Navier-Stokes equations in vector form:

(5) $$\frac{|v.\nabla v|}{2|\Omega \times v|}(\frac{\partial v}{\partial t} + v.\nabla v = -\frac{1}{\rho}\nabla P - \Omega \times (\Omega \times r) - 2\Omega \times v + \nu \nabla^2 v)$$

Where, it is used the well known basic denotations as follows: ρ - is the mass density of the flow; v - is the velocity of the flow; P - is the pressure; ν - is the kinematic viscousity; Ω - is the angular velocity; $\Omega \times (\Omega \times r)$ - is the centrifugal force of the rotating accretion flow; $2\Omega \times v$ - expresses the Coriolis force;

After implying an implicit differentiation, it is yielded the velocity distribution and its graphic form is in the figure 1:

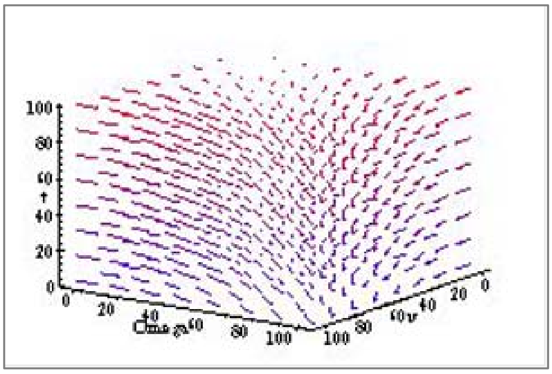


***Fig. 2**. The distribution shows that at the high values of parameters the dynamical processes are more intensive in the presence of non-integrated angular velocity*

Even at low values, it is obviously that the Rossby number gives rise to velocity variations. This fact shows us that we should not neglect its presence, when we study the behaviour and dynamics of accretion disc flow.

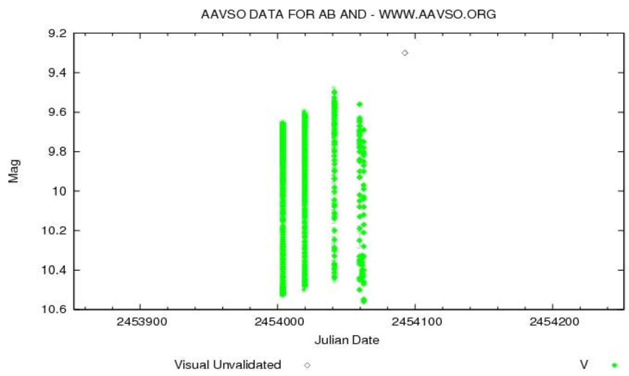
Results C

The third result of our study, which we present here, shows the indirect effect of Rossby number on the shape of light curves.

The variations of Rossby number could be contribute in the process of some vortex formations in the accretion disc area. Its increment acts as an accelerator of the transition from laminar to turbulent motion of the flow. As we consider binary star system with accretion disc, we show the light curve of AB And produced at such unstable states. The shape of light curve gives us information about the strong disturbances with the suddenly appearing bursts.

The presence of an accretion disc affects on the light curve in two ways: as a source of light and as a shield of the surface of the part of the companion from the X-ray heating.

It is detected that in depends of Rossby number the lightcurve produces the variation in its shape. It acts usually at small value and there is a gradual, broad modulation. Whereas, it begins to see the effects of two-spiral arm emitting region and the narrow peaks of the brightness. Such lightcurve modulations may associate with Rossby type of instability



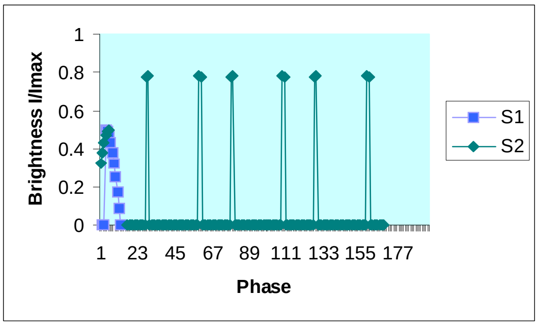


Fig. 3. *Light curve of AB AND binary star. The two figures present the creating of graphic by two different programs and methods.*
1. *Using AAVSO data /left panel/.* ***2.*** *Preparing by program of Kreiner [2004] /right panel/*

It is seen in the figures the brightness peaks, result of hard outbursts. The above selected types of binaries are important for us, because of their high inclination (i). As it is mention above, this feature is an indicator for arising of unstable processes, such as wave propagation, structure formation and Rossby type of instability.

III. Conclusion

From an astrophysical point of view, it is important to investigate the turbulence and vorticity formations as a main transport mechanism of angular momentum in accretion discs.
Based on the properties of differentially rotating flow in the accretion disc area, we emphasized in our analyses on the presence of special relation between inertial and Coriolis force, known as a Rossby number. We considered the accretion disc as a hydrodynamical system and we used the base equation of Navier-Stokes, including Rossby number. The results show the effect of this number on several situations: the effect over the velocity distribution; the range of Rossby number values which are effective for our astrophysical study; influence over the variations of light curve of selected binary star.
Remembering that high (resp. low) Rossby number are associated with three-(resp. two-) dimensional motions, this result is coherent with the classic picture of a turbulent spectrum showing two- dimensional structures at large scales and three-dimensional ones at smaller scales.
We presented in this survey our common results of the studying problem, without specify the values of the Rossby number for different phenomena or for a star.

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