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NONLINEAR DYNAMICAL PROCESSES IN EXTRA-SOLAR PLANETARY SYSTEMS

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Abstract: *Динамиката на системи от три или повече тела обикновено включва физически процеси, известни като резонанси на средното движение и секулярни пертурбации. Първите се появяват, когато двойка тела имат орбитални периоди, чието съотношение може приблизително да се представи като съотношение на две малки цели числа. Вторите са съществен фактор при изследване на дълговремевата еволюция на системата. Динамичната еволюция на над половината от известните планетите в мулти-планетарните извънслънчеви системи се доминира от секулярните резонанси. Най-често големите ексцентрицитети на планетарните орбити поставят под съмнение полезността на традиционната секулярна теория на Лагранж-Лаплас при анализа на движение. Тази теория може да бъде обобщена до четвърти порядък в ексцентрицитета, след което да се сравнява с числените резултати. Част от изводите, до които се стига в резултат на тези сравнения, са - Лагранж-Лапласовата теория на секулярната динамика е слаб индикатор (инструмент) за предсказване на секулярната динамима на извънслънчеви планетарни системи, но е полезен инструмент при прецизното определяне на дълговремевата динамична еволюция на системи от малки тела с орбити, близки до кръговите*.

Secular Interactions - Basic Theory

Here we outline the basic theory of secular interactions as applied to the planetary systems. Since this topic has been well studied, the review and the results are brief. .

The equations of motion for eccentricity e_j (to the second order in eccentricity and inclination angle) and argument of periastron ϖ_j decouple from those of inclination angle and the ascending node. Following standard convention (Murray & Dermott 1999), we work with the variables defined by:

(1) $$h_i \equiv e_i \sin\varpi_i \quad and \quad k_i \equiv e_i \cos\varpi_i \ ,$$

where the subscript i refers to the i-th planet in an N planet system. The basic equations of motion for the theory can then be written in the form

(2) $$\frac{dh_i}{dt} = \frac{1}{n_i a_i^2}\frac{\partial R_i}{\partial k_i} \quad and \quad \frac{dk_i}{dt} = -\frac{1}{n_i a_i^2}\frac{\partial R_i}{\partial h_i} \, ,$$

where R_i is the secular part of the disturbing function, n_i is the mean motion of the i-th planet and a_i is its corresponding semi-major axis. To consistent order in this approximation, the relevant terms in the disturbing function take the form

(3) $$R_i = n_i a_i^2 \left[\frac{1}{2} A_{ii} e_i^2 + \sum_{k \neq i} A_{ik} e_i e_k \cos(\varpi_i - \varpi_k) \right]$$

The physics of these interactions is thus encapsulated in the $N \times N$ matrix A_{ij}, where the number N of planets in the system is usually N = 2 or 3 for the systems observed, to date. The matrix elements can be written in the form

(4) $$A_{ii} = n_i \left[\frac{1}{4} \sum_{k \neq i} \frac{m_k}{M_* + m_i} \alpha_{ik} \overline{\alpha}_{ik} b_{3/2}^{(1)}(\alpha_{ik}) + 3\frac{GM_*}{c^2 a_i} \right]$$

and

$$A_{ik} = -n_i \frac{1}{4} \frac{m_k}{M_* + m_i} \alpha_{ik} \overline{\alpha}_{ik} b_{3/2}^{(2)}(\alpha_{ik}). \quad (5)$$

In the diagonal matrix elements (eq. [4]) it is included the leading order correction for general relativity (***c*** is the speed of light). Although these terms are small, $\mu \equiv \frac{GM_*}{c^2 a_i} << 1$, such small corrections to the eigenfrequencies can be important, especially when the system is near resonance. The quantities α_{ik} are defined such that $\alpha_{ik} = a_i / a_k (a_k / a_i)$ if $a_i < a_k (a_k < a_i)$. The complementary quantities $\overline{\alpha}_{ik}$ are defined so that $\overline{\alpha}_{ik} = a_i / a_k = \alpha_{ik}$ if $a_i < a_k$, but $\overline{\alpha}_{ik} = 1$ for $a_k < a_i$. Finally, the quantities $b_{3/2}^{(1)}(\alpha_{ik})$ and $b_{3/2}^{(2)}(\alpha_{ik})$ are Laplace coefficients.

With the above definitions, the resulting solution takes the form

$$h_j = \sum_i \Lambda_{ji} \sin(\lambda_i t + \beta_i), \quad k_j = \sum_i \Lambda_{ji} \cos(\lambda_i t + \beta_i), \quad (6)$$

where the λ_i are eigenvalues of the matrix A_{ij} and the Λ_{ji} are the corresponding eigenvectors. The phases β_i and the normalization of the eigenvectors are determined by the initial conditions, i.e., the values of eccentricity e_j and argument of periastron ϖ_j for each planet at $t = 0$.

Applications to Extrasolar Planetary Systems. Eccentricity Distributions and Secular Time Scales

We use the theory of secular interactions to show the relationship between the observed values of eccentricity and the underlying distribution of eccentricities that characterize the systems. We use interactions, and their absence, to place new constraints on observed multiple planet systems. We can find constraints on the possible existence of additional small (terrestrial) planets in these systems by requiring that any such planets must reside far from a secular resonance. As an application of secular interaction theory, we use the formalism described above to calculate the variations in eccentricity in a sub-sample of observed extrasolar planetary systems. The eigenvalues λ_i for the multiple planet systems calculate and convert into time scales for a collection of 16 observed multiple planet systems. Most of these multiple planet systems have secular interaction time scales in the range $10^3 - 10^5$ yr. These time scales are much longer than any possible observational baseline (tens of years), but much shorter than the system lifetimes (which are typically several Gyr). The shortest secular time scale occurs for the GJ 876 system. Although the dynamics of this system are dominated by the 2:1 resonance between planets "*c*" and "*b*", the secular interaction time $\tau = 4.4$ yr gives an excellent estimate of the time scale for dynamical interaction in the system. Indeed, radial velocity fits to the system must take into account the planet-planet interactions in order to obtain an acceptable fit.

These secular interaction times are thus long enough that observations can determine the eccentricity (and longitude of periastron) with high accuracy at the present epoch. Over much longer time scales that are not observationally accessible, however, the eccentricity (and longitude of periastron) will vary according to the appropriate secular cycles. As a result, attempts to explain the observed (*a, e*) plane must take the possibility of secular variations into account. The effect of secular interactions on the observational interpretation of these systems is that the measured eccentricity values are a particular sampling of an underlying distribution. Within the context of leading order secular theory, the distribution of eccentricity is determined by the above formalism. For each of the observed multiple planet systems considered here calculates the expected time variations of eccentricity and longitude of periastron according to secular theory. From this time series were extracted the mean eccentricity $\langle e \rangle$, the variance σ_e of the distribution, the minimum eccentricity value $e_{\min}$ and the maximum value $e_{\max}$. The difference between the observed values and the mean eccentricities averaged over many secular cycles can be substantial (more than a factor of two). The width of the distribution can also be significant.

For the case of two planet systems, the formalism produces simple analytic expressions for the parameters of the eccentricity distribution. The distribution itself can be derived by taking the solution of equation (6) and solving for the eccentricity as a function of time. Since time is distributed uniformly, the resulting expression can be solved for the corresponding distribution of eccentricity, which can then be written in the form

(7) $$\frac{dP}{de} = \sqrt{1 - \left(\frac{e^2 - \Lambda_{j1}^2 - \Lambda_{j2}^2}{2\Lambda_{j1}\Lambda_{j2}} \right)^2} \frac{e}{\pi\Lambda_{j1}\Lambda_{j2}},$$

where Λ_{ji} are the eigenvectors. This form of the eccentricity distribution (for the *i*-th planet) is valid between the extremes given by

(8) $$e_{\max}, e_{\min} = \left| \Lambda_{j1} \pm \Lambda_{j2} \right| .$$

The mean value of the distribution can be evaluated from its definition $\langle e \rangle = \int e(dP/de)de$ and takes the form

(9) $$\langle e \rangle_j = \frac{2}{\pi}(\Lambda_{j1} + \Lambda_{j2})E(\hat{m}),$$

Where $E(\hat{m})$ is the elliptical integral of the second kind (Abramowitz & Stegun, 1970) with parameter $\hat{m} = 4\frac{\Lambda_{j1}\Lambda_{j2}}{(\Lambda_{j1} + \Lambda_{j2})^2}$. Notice that the parameter $\hat{m}$ can be negative and hence care must be taken in evaluating $E(\hat{m})$. The corresponding variance of the distribution is given by

(10) $$\sigma_{ej}^2 = \Lambda_{j1}^2 + \Lambda_{j2}^2 - \frac{4}{\pi^2}(\Lambda_{j1} + \Lambda_{j2})^2 [E(\hat{m})]^2$$

For two-planet systems, it has verified that these expressions for the mean, extrema, and variance of the distribution are in good agreement with those found via sampling of the secular solutions as described above.

Conclusion

Through dynamical interactions, described here using secular theory, the orbital eccentricities in multiple planet systems vary over secular time scales. The eccentricities measured by ongoing planet searches represent the current eccentricity value, which is drawn from a wider distribution of values sampled by the planet. In other words, the eccentricities in multiple planet systems should not be considered as particular values, but rather as distributions of values. The widths of these eccentricity distributions can be substantial and it has verified that secular theory predicts distribution widths that are in good agreement with direct numerical integration. For the simplest case of two planet systems, the resulting distribution of eccentricity can be found analytically (eqs. [7 – 10]). The time scale for secular eccentricity variations is typically thousands of, much longer than observational survey time scales (tens of years) and much shorter than the system lifetimes (few Gyr). Secular interactions can add to our understanding of these forthcoming multiple planet systems in a variety of ways. In trying to find theoretical explanations for the observed orbital elements, one must take into account the distributions of eccentricities driven by secular interactions. In systems with known giant planets, the search for Earths can be guided by studying the forced eccentricity variations. In other systems, we can deduce the presence or absence of additional (undetected) planets — or at least constrain their properties — through examination of the properties of the detected planets. Over longer time spans, secular interactions combine with tidal circularization and energy dissipation processes.

References:

1. Thommes E.W., M.J. Duncan & H.F. Levison. 2002, AJ, 123, 2862.
2. Thommes E.W., M.J. Duncan and H.F. Levison. 1999, Nature, 402, 635.
3. Weidenschillingm S.J. and Marzari F. 1996, Nature, 384, 619.
4. Wisdom J. and Holman M. 1991, AJ, 102, 1528.
5. Levison H.F., J.J. Lissuaer, M.J. Duncan. 1998, AJ, 116, 1998.
6. Murray N., B. Hansen, M. Holman, S. Tremaine. 1998, Science 279, 69.
7. Morbidelli A., H.F. Levison, K. Tsiganis, R. Gomes. 2005, Nature, 435, 462.
8. Morbidelli A. 2002, Modern Celestial Mechanics: Aspects of Solar System Dynamics (London and New York).
9. Tsiganis K., R. Gomes, A. Morbidelli and H.F. Levison. 2005, Nature, 435, 459.
10. Michtchenko T.A., R. Malhotra. 2004, Icarus, 168, 237.
11. Mikkola S. 1997, CeMDA, 67, 145.
12. Duncan M.J., T. Quinn. 1993, ARAA, 31, 265.